ONSET OF OSCILLATION IN THE PRESENCE OF DETONATION WAVE WEAKENING

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The weakening of supercompressed detonation waves by rarefaction waves approaching these from behind was considered in a number of papers. It was established in [1] that the interaction of an infinitely narrow plane supercompressed detonation wave with a sufficiently intensive rarefaction flow behind it results in a gradual weakening of the wave, and its asymptotic transition to the Chapman-Jouguet mode. With weakening cylindrical and spherical supercompressed waves this transition to the Chapman-Jouguet mode may occur at a finite distance, not asymptotically as is the case of plane waves [2]. The asymptotic behavior of a plane detonation wave of the "two-front" pattern, i.e. of a detonation wave consisting of an adiabatic jump followed by a heat release jump, the distance between the two jumps being dependent on the time elapsed between the instant a gas particle passes through the first jump and the instant of its ignition. It was shown, with a number of simplifying assumptions (see below), that the transition of such wave to the Chapman-Jouguet pattern occurs at sufficiently small values of the activation energy only. When the activation energy is high, a small variation of initial conditions leads to an exponential deviation of the wave front from the asymptotic path corresponding to the Chapman-Jouguet pattern.

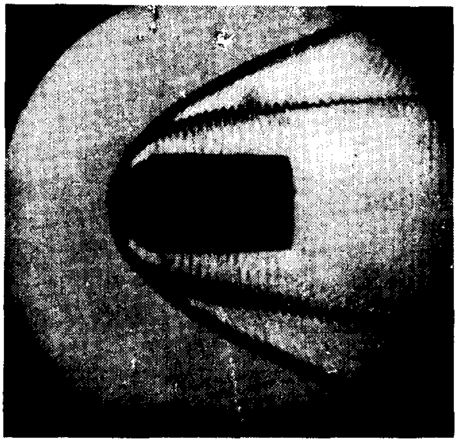

Fig. 1

The problem of the asymptotic behavior of weakened detonation waves is closely related to that of the stability of such waves with respect to various perturbations. The instability of a plane two-front detonation waves associated with a possible spontaneous heat front disintegration was considered in [4, 5]. Paper [4] had in particular established a criterion according to which this type of instability occurs when the activation energy is sufficiently high.

The instability occurring in weakened detonation waves may lead to oscillations of considerable amplitude in the stream of gas, and to the disintegration of detonation waves. An example of such detonation wave disintegration in front of a body flying at high velocity in a hydrogen-air mixture is shown in Fig. 1 (*).

A detailed analysis is made in this paper of the problem of weakening of a plane detonation wave of the two-front type by rarefaction perturbations approaching it from

*) Photograph taken at the Institute of Mechanics of the Moscow State University by V. V. Semchenko.

behind; the oscillation initiation mechanism is investigated, and the criterion of wave stability behavior when subject to weakening and in transition to the Chapman-Jouguet mode is estabilshed.

We shall consider a one-dimensional flow of gas in the presence of a two-front detonation wave (Fig. 2). Let v, p and ρ be respectively the gas velocity, pressure and density. Variation of these parameters in each of the three regions, separated from each other by the adiabatic jump front s and the heat release front f are defined by the equations of adiabatic motion of gas. We shall denote by subscript ∞ the parameters upstream of the adiabatic jump, and by subscript 1 those between the two fronts, while leaving the magnitudes downstream of the heat release wave without a subscript.

The gas parameters must be bound by conditions

at the adiabatic jump

$$v_1 - v_\infty = \frac{2}{\gamma+1}\left[\frac{a_\infty^2}{v_\infty - c_s} - (v_\infty - c_s)\right] \tag{1}$$

$$p_1 = p_\infty + \rho_\infty (v_\infty - c_s)(v_\infty - v_1), \qquad \rho_1 = \frac{\rho_\infty (v_\infty - c_s)}{v_1 - c_s}$$

and at heat release front

$$v - v_1 = \frac{1}{\gamma+1}\left\{\frac{a_1^2}{v_1 - c_f} - (v_1 - c_f) - \left(\left[\frac{a_1^2}{v_1 - c_f} - (v_1 - c_f)\right]^2 - 2(\gamma^2 - 1)Q\right)^{1/2}\right\}$$

$$p = p_1 + \rho_1 (v_1 - c_f)(v_1 - v), \qquad \rho = \frac{\rho_1 (v_1 - c_f)}{v - c_f} \tag{2}$$

Here a is the velocity of sound, while c_s and c_f are respectively the propagation velocities of the adiabatic jump, and of the heat release front. The gas is assumed to be perfect and its specific heat ratio γ constant throughout the whole stream. The heat release Q per mass unit of gas passing through front f is assumed to be given.

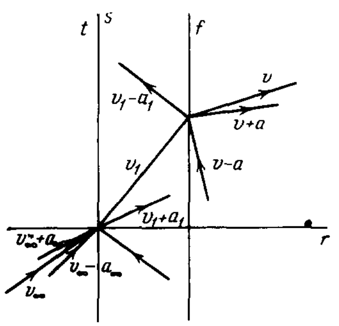

Fig. 2

The heat release front propagates through the gas at subsonic velocity (to which corresponds the selected minus sign in front of the radical of the first condition of (2), hence, from it emanate the characteristics of three sets, one of which is directed upstream. Because of this, conditions (2) are insufficient for a unique determination of perturbations moving away from the front along the characteristics, and of the front velocity from known perturbations approaching the front, and must be supplemented by one more condition. For the problem here considered this condition is given by the specified ignition time lag τ, i.e. the time it takes a gas particle to pass from the adiabatic to the heat release front. In fact, because along the particle trajectory

$$r_f(t+\tau) = r_s(t) + \int_t^{t+\tau} v\,dt \tag{3}$$

hence, for a given τ this equation provides the supplementary condition necessary for the unique definition of the perturbation moving upstream and away from the heat release front. In order to allow for the effect of thermodynamic conditions to which the particle is subjected on time τ, we shall determine this time from the following relationship taken along the particle trajectory after it had passed through the adiabatic jump

$$\int_{t}^{t+\tau} f(p, T)\,dt = 1 \tag{4}$$

If the particle pressure and temperature T remain unchanged, then time τ is simply equal to $1 / f(p, T)$. Function f may, e.g., be presented in the form

$$f = kp^{m-1} \exp\left(\frac{-E}{RT}\right) \tag{5}$$

where E is the activation energy, R the gas constant, and m and k are constants.

As the initial state we shall take the stationary wave structure in a system of coordinates in which the wave is at rest. All magnitudes appertaining to this state will be denoted by subscript 0. We shall consider the nonstationary motions produced by the interaction of this wave with perturbations approaching it from behind.

On the assumption of a weakly perturbed motion in the layer between the two fronts, we represent the gas parameters in this layer in the form

$$v_1 = v_{10} + \delta v_1, \qquad p_1 = p_{10} + \delta p_1, \qquad \rho_1 = \rho_{10} + \delta\rho_1$$

From equations of gas dynamics we derive the linear approximation

$$\delta v_1 = v_{10}(F + G), \quad \delta p_1 = \rho_{10} v_{10} a_{10}(-F + G),$$
$$\delta\rho_1 = \frac{\rho_{10} v_{10}}{a_{10}}(-F + G) + \rho_{10} H \tag{6}$$

where each of the functions F, G and H depend respectively on one of the characteristic arguments $\xi = r - (v_{10} - a_{10})\,t, \quad \eta = r - (v_{10} + a_{10})\,t, \quad \zeta = r - v_{10} t$

Conditions (1) along the adiabatic front yield after linearization the following relationships between functions F, G and H at that front (in a linear approximation with $r = 0$), and the velocity c_s of the latter:

$$G_s = -\lambda F_s, \quad H_s = \sigma F_s, \quad \frac{c_s}{v_{10}} = \varkappa F_s \tag{7}$$

Here λ, σ and $\varkappa$ are functions of the Mach number M_∞ of the oncoming stream and of the specific heat ratio γ

$$\lambda = \frac{2M_\infty^2 M_{10} - 1 - M_\infty^2}{2M_\infty^2 M_{10} + 1 + M_\infty^2}, \quad \sigma = \frac{4(M_\infty^2 M_{10}^2 - 1)}{2M_\infty^2 M_{10} + 1 + M_\infty^2},$$
$$\varkappa = \frac{(\gamma + 1) M_\infty^2}{2M_\infty^2 M_{10} + 1 + M_\infty^2}$$

The relation between M_{10} and M_∞ is defined by formula

$$M_{10}^2 = \frac{1 + {}^1\!/_2(\gamma - 1) M_\infty^2}{\gamma M_\infty^2 - {}^1\!/_2(\gamma - 1)}$$

Magnitude $\lambda = -G_s / F_s$ provides a standard for the variation of pressure perturbation at its reflection from the jump, and is usually referred to as the coefficient of weak perturbation reflection from a jump.

We emphasize that the specification of perturbations approaching the jump from behind along characteristics $\xi = \text{const}$ (i.e. function F) completely defines the perturbations moving away from the jump along characteristics $\eta = \text{const}$ and $\zeta = \text{const}$ (i.e. functions G and H) and also the jump velocity c_s.

From (3) and (4) we obtain in a linear approximation (assuming that τ varies insignificantly with varying p and T)

$$r_f(t+\tau_0) = r_s(t) + v_{10}\tau_0 + v_{10}\int\limits_t^{t+\tau_0}\left(\frac{\delta v_1}{v_{10}} - \frac{\partial \ln f}{\partial \ln p}\Big|_0 \frac{\delta p_1}{p_{10}} - \frac{\partial \ln f}{\partial \ln T}\Big|_0 \frac{\delta T_1}{T_{10}}\right) dt \quad (8)$$

This expression has been derived earlier in [4].

Using formulas (6) we integrate, and present expression (8) in the form

$$r_f(t+\tau_0) = r_s(t) + v_{10}\tau_0 + v_{10}\int\limits_t^{t+\tau_0} [(1+\mu M_{10})F(a_{10}t' - v_{10}t) +$$
$$+ (1-\mu M_{10})G(-a_{10}t' - v_{10}t)]\,dt' - nH(-v_{10}t)\,v_{10}\tau_0 \quad (9)$$

Here

$$\mu = (\gamma - 1)n + \gamma m', \quad n = \frac{\partial \ln f}{\partial \ln T}\Big|_0, \quad m' = \frac{\partial \ln f}{\partial \ln p}\Big|_0$$

When function f is defined by (5), then

$$n = \frac{E}{RT_{10}}, \qquad m' = m - 1$$

Differentiating Eq. (9) with respect to t we obtain the following condition:

$$\frac{c_{f,\,t+\tau_0}}{v_{10}} = \frac{c_{s,\,t}}{v_{10}} + b_1(F_{f,\,t+\tau_0} - F_{s,t}) + b_2(G_{f,\,t+\tau_0} - G_{s,t}) - nv_{10}\tau_0 H_{s,t} \quad (10)$$

where

$$b_1 = (1 - M_{10})(1 + \mu M_{10}), \qquad b_2 = (1 + M_{10})(1 - \mu M_{10})$$

Condition (10) binds the gas parameter values of one of the same particle when this is at a point behind the adiabatic jump, and in front of the heat release front. Noting that parameters c_s, F_s and G_s can be expressed in terms of H_s by using relationships (7), and that function H holds when subscripts f and $t + \tau_0$ are substituted for s and t, we can readily transform condition (10) in such a way that it will contain parameters of the gas in front of the heat release front and of that front velocity only (subscript f is here, and in the following omitted)

$$\frac{c}{v_{10}} = b_1 F + b_2 G + \frac{\varkappa - b_1 + b_2\lambda}{\sigma} H - nv_{10}\tau_0 H' \quad (11)$$

We shall now revert to conditions (2) binding the gas parameters on the two sides of the heat release front. We introduce parameter Δ which defines the difference between an unperturbed detonation wave and the Chapman-Jouguet wave by means of the formula

$$\left(\frac{a_{10}^2}{v_{10}} - v_{10}\right)^2 - 2(\gamma^2 - 1)Q = v_J^2\Delta^2$$

Then

$$v_0 = \frac{1}{\gamma+1}\left(\gamma v_{10} + \frac{a_{10}^2}{v_{10}} - v_J\Delta\right) = v_J\left(1 - \frac{\Delta}{\gamma+1}\right), \quad v_J = \frac{1}{\gamma+1}\left(\gamma v_{10} + \frac{a_{10}^2}{v_{10}}\right)$$

$$p_0 = p_{10} + \rho_{10}v_{10}(v_{10} - v_0) = p_J + \rho_{10}v_{10}v_J\frac{\Delta}{\gamma+1}$$

$$\rho_0 = \frac{\rho_{10}v_{10}}{v_0} = \frac{\rho_J}{1 - \Delta/(\gamma+1)}, \qquad a_0^2 - v_0^2 = v_0 v_J \Delta$$

Relationships (2) for $r = v_{10}\tau_0$ are for any Δ and small perturbations of the form

$$\delta v = v - v_0 = \frac{1}{\gamma+1}\left\{\left(1 + \frac{1}{M_{10}^2}\right)c + \left(\gamma - \frac{1}{M_{10}^2} - \frac{\gamma-1}{M_{10}}\right)v_{10}F +\right.$$
$$+ \left(\gamma - \frac{1}{M_{10}^2} + \frac{\gamma-1}{M_{10}}\right)v_{10}G - \frac{1}{M_{10}^2}v_{10}H + v_J\Delta - \quad (12)$$

(cont.)

$$-\left[v_J^2\Delta^2 + 2v_{10}\left(\frac{1}{M_{10}^2}-1\right)\left[\left(1+\frac{1}{M_{10}^2}\right)c - \left(1+\frac{1}{M_{10}^2}+\frac{\gamma-1}{M_{10}}\right)v_{10}F - \right.\right.$$
$$\left.\left.-\left(1+\frac{1}{M_{10}^2}-\frac{\gamma-1}{M_{10}}\right)v_{10}G - \frac{1}{M_{10}^2}v_{10}H\right]\right]^{1/2}\Bigg\}$$

$$\frac{\delta p}{\rho_0 v_0} = -\delta v - \left(1-\frac{v_0}{v_{10}}\right)c + \left[2-\frac{1}{M_{10}}-M_{10}-(1-M_{10})\frac{v_0}{v_{10}}\right]v_{10}F +$$
$$+\left[2+\frac{1}{M_{10}}+M_{10}-(1+M_{10})\frac{v_0}{v_{10}}\right]v_{10}G + v_{10}\left(1-\frac{v_0}{v_{10}}\right)H$$
$$\frac{\delta\rho}{\rho_0} = -\frac{\delta v}{v_0} + \left(\frac{v_{10}}{v_0}-1\right)\frac{c}{v_{10}} + (1-M_{10})F + (1+M_{10})G + H$$

In the beginning we shall assume that parameter Δ is not small, i.e. that the original detonation wave is not close to the Chapman-Jouguet mode. Then the parameters c/v_{10}, F, G and H in the relationships (12) may be considered small in comparison to Δ^2, and we can write the expression of the flow downstream of the heat release front in a linear approximation, i.e. by analogy to expressions (6) as follows:

$$\delta v = v_0(F_* + G_*), \quad \delta p = \rho_0 v_0 a_0(-F_* + G_*)$$
$$\delta\rho = \frac{\rho_0 v_0}{a_0}(-F_* + G_*) + \rho_0 H_*$$

After linearization of the radical in the first of Eqs. (12), its three conditions together with condition (11) yield four linear equations relating the heat release front velocity c_f and the three perturbations F, G_* and H_* moving away from the front to the three perturbations G, H and F_* approaching the front. For the determination of c_f and F we have in addition to (11) the following relationship:

$$\left\{1-\frac{v_0}{v_{10}}+\frac{1}{\gamma+1}\left(\frac{1}{M_0}+1\right)\left(1+\frac{1}{M_{10}^2}\right)\left[1-\frac{v_{10}}{v_J\Delta}\left(\frac{1}{M_{10}^2}-1\right)\right]\right\}c +$$
$$+\left\{(1-M_{10})\frac{v_0}{v_{10}} - 2 + \frac{1}{M_{10}} + M_{10} + \frac{1}{\gamma+1}\left(\frac{1}{M_0}+1\right)\left[\gamma - \frac{1}{M_{10}^2} - \frac{\gamma-1}{M_{10}} + \right.\right.$$
$$\left.\left.+\frac{v_{10}}{v_J\Delta}\left(\frac{1}{M_{10}^2}-1\right)\left(1+\frac{1}{M_{10}^2}+\frac{\gamma-1}{M_{10}}\right)\right]\right\}v_{10}F + \left\{(1+M_{10})\frac{v_0}{v_{10}} - 2 - \frac{1}{M_{10}} - \right.$$
$$-M_{10} + \frac{1}{\gamma+1}\left(\frac{1}{M_0}+1\right)\left[\gamma - \frac{1}{M_{10}^2} + \frac{\gamma-1}{M_{10}} + \right. \tag{13}$$
$$\left.\left.+\frac{v_{10}}{v_J\Delta}\left(\frac{1}{M_{10}^2}-1\right)\left(1+\frac{1}{M_{10}^2}-\frac{\gamma-1}{M_{10}}\right)\right]\right\}v_{10}G -$$
$$-\left\{1-\frac{v_0}{v_{10}}+\frac{1}{\gamma+1}\left(\frac{1}{M_0}+1\right)\frac{1}{M_{10}^2}\left[1-\frac{v_{10}}{v_J\Delta}\left(\frac{1}{M_{10}^2}-1\right)\right]\right\}v_{10}H - 2a_0F_* = 0$$

In the absence of perturbations approaching the heat release front, i.e. when $G = H = F_* = 0$, Eqs. (11) and (13) yield two different relationships binding the heat release front velocity c with perturbation F moving upstream and away from it. Condition (11) which is the consequence of the presence of a definite ignition time lag may be called "the chemical" condition, and condition (13) which follows from the laws of conservation at the heat release front may be called "the gas-dynamical" condition.

When no perturbations approach the heat release front, then its velocity $c^{(1)}$ is, in accordance with the chemical condition, related to magnitude F by relationship

$$c^{(1)} = b_1 v_{10} F$$

Velocity $c^{(2)}$, when defined for the same values of F by the gasdynamical condition, is

$$c^{(2)} = B_1 v_{10} F$$

where B_1 is to be obtained from Eq. (13). According to [4] a two-front detonation wave is unstable for $(c^{(1)}/c^{(2)}) > 1$, and stable in the opposite case. The fulfilment of this instability criterion is an indication that in the absence of perturbations there exists in addition to the solution in which the release front is stationary, another solution in which this front disintegrates, moves at a new velocity, and emits shock and centered rarefaction waves. (If a rarefaction wave moving upstream is generated at the disintegration, then the motion will be self-similar only on condition that function $f\ (p,\ T)$ in Eq. (4) does not contain constants which would make possible the formation of time, or length scales from the defining magnitudes). Generally, solutions containing discontinuity disintegration will not be close to the initial unperturbed state. Transition from the initial stationary state to the solution with discontinuity disintegration is essentially a nonlinear process. A number of solutions containing discontinuity disintegration was analyzed in [5].

The stability boundary of the ignition time lag, as determined from formula (5) is shown in Fig. 3 in terms of dependence of E / RT_∞ on M_∞ and $\bar{Q} = Q / c_p T_\infty$ (computed for $\gamma = 1.4$ and $m = 1$).

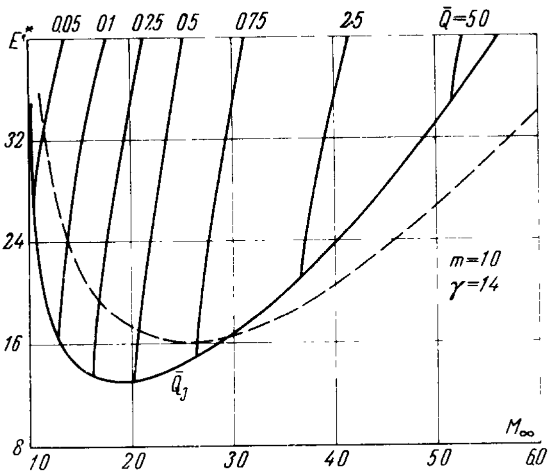

Fig. 3

We would point out that formulas (13) may obviously be used in the determination of the stability boundary also when Δ is arbitrarily small. It follows from Fig. 3 that for considerable values of activation energy E and fixed heat release rate Q the detonation wave becomes unstable when approaching the Chapman-Jouguet mode.

Instability of the detonation wave considered in [7] may be termed inner instability. As already stated, its character is nonlinear, and it may occur spontaneously.

In the absence of perturbations reaching the heat release front Eqs. (2) and (4) in linear approximation have solutions different from the solution free of perturbations only at the stability limit defined above. In all other cases instability may only occur as the result of a gradual development of a perturbation acting on the wave stationary structure.

We revert to Eqs. (11) and (13), and assume that the detonation wave interacts with the perturbation reaching it from behind. In this case perturbations G and H approaching the heat release front from upstream will be reflections from the adiabatic jump of perturbations F moving forward from the heat release front, i.e.

$$G_{f,\,t} = -\lambda F_{f,\,t-\delta_1}, \quad H_{f,\,t} = \sigma F_{f,\,t-\delta_2}, \quad H_{f,\,t}' = -\sigma\left(\frac{a_{10}}{v_{10}} - 1\right) F_{f,\,t-\delta_2}'$$

where

$$\delta_1 = \frac{2M_{10}}{1 - M_{10}^2}\tau_0, \quad \delta_2 = \frac{\tau_0}{1 - M_{10}}$$

Using these relationships, and substituting for c^f in Eq. (13) its expression given by (11), we obtain for function F the following equation:

$$F_{f,t} = \lambda' F_{f,\,t-\delta_1} + \lambda'' F_{f,\,t-\delta_2} + \lambda''' v_{10}\tau_0 F'_{f,\,t-\delta_2} + K\Delta F_{*f,\,t} \qquad (14)$$

where parameters $\lambda', \lambda'', \lambda'''$ and K are linear-fractional functions of m' and n with coefficients of known dependence on the undisturbed flow parameters. For simplicity's sake we shall adduce the expressions of these parameters for $\Delta \ll 1$ only, i.e. for detonation modes close to the Chapman-Jouguet mode

$$\lambda' = \frac{1 + M_{10}^2 - (\gamma - 1) M_{10} - (1 + M_{10}^2) b_2}{1 + M_{10}^2 + (\gamma - 1) M_{10} - (1 + M_{10}^2) b_1}\lambda, \quad \lambda'' = \frac{(\varkappa - \lambda b_1 + \lambda b_2)(1 + M_{10}^2) - \sigma}{1 + M_{10}^2 + (\gamma - 1) M_{10} - (1 + M_{10}^2) b_1}$$

$$\lambda''' = \frac{\sigma (1 + M_{10}^2)(M_{10}^{-1} - 1)\, n}{1 + M_{10}^2 + (\gamma - 1) M_{10} - (1 + M_{10}^2) b_1}$$

$$K = \frac{(1 + \gamma M_{10}^2)^2}{(\gamma + 1)(1 - M_{10}^2)[1 + M_{10}^2 + (\gamma - 1) M_{10} - (1 + M_{10}^2) b_1]}$$

All of these magnitudes increase, as they should, indefinitely when approaching the stability limit, as was shown in [4]. In the limit case considered here this limit is defined by the formula

$$\mu = \frac{\gamma + M_{10}^2}{(1 + M_{10}^2)(1 - M_{10})}$$

The boundary curve $\overline{Q} = \overline{Q}_J$, or $\Delta = 0$, of Fig. 3 correspond to this limit.

We recall that Eq. (14) with coefficients defined by formulas (15) holds for $F \ll \Delta^2$ only, hence condition $F_* \ll \Delta$ must be fulfilled for incident perturbation F_*.

It is, however, interesting to consider incident perturbations of the order of Δ. This is particularly so, because in the analysis of a slow and weak supercompressed detonation wave transition to the Chapman-Jouguet mode the perturbation which weakens the detonation wave must be of this order.

We revert to Eqs. (12), and shall consider the case of a perturbation of order Δ incident on the heat release front. From the first condition of (12) follows that c_f / v_{10} and F (and consequently in this problem also functions G and H) must be of the order of Δ^2. Omitting terms of higher order, we derive from the first condition of (12)

$$v - v_J = -\Big\{v_J^2\Delta^2 + 2v_{10}^2\left(\frac{1}{M_{10}^2} - 1\right)\Big[\left(1 + \frac{1}{M_{10}^2}\right)c - \\ -\left(1 + \frac{1}{M_{10}^2} + \frac{\gamma - 1}{M_{10}^2}\right)F - \left(1 + \frac{1}{M_{10}^2} - \frac{\gamma - 1}{M_{10}}\right)G - \frac{1}{M_{10}^2}H\Big\}^{1/2} \qquad (15)$$

In accord with the results presented in [1] (or by a direct substitution of this expression of v into the remaining conditions (12)) we conclude that downstream of the heat release front perturbations of the order of Δ represent a simple wave reaching the heat release front, while perturbations moving downstream away from this front are of the order of Δ^2.

Thus, the left side of expression (15) should be considered as given, and consequently, when the perturbations reaching the wave are of the order of Δ, this condition replaces condition (13) valid when the approaching perturbations are of a higher order.

It will be readily seen that condition (13) in the limit case of small Δ and condition (15) differ only as regards the terms defining the perturbations approaching the front from behind. Hence, we can use in this case, as previously, Eq. (14) with the same values of coefficients λ', λ'' and λ''', but with the substitution in its last tern of magnitude

$$\frac{1}{2(\gamma+1)}\left[\Delta^2-(\gamma+1)^2\frac{(v_J-v)^2}{v_J^2}\right] \tag{16}$$

for $\Delta F_{*f,t}$.

We shall now consider the simple unperturbed wave approaching the heat release front from behind. The formulas defining such a wave

$$r=(v-a)\,t+\Phi(v),\qquad v+\frac{2a}{\gamma-1}=\text{const}$$

may be replaced in the approximation here considered by the following:

$$r=\frac{\gamma+1}{2}(v-v_J)\,t+\Phi(v),\qquad v+\frac{2a}{\gamma-1}=v_J+\frac{2a_J}{\gamma-1}$$

If function $\Phi(v)$ has a finite derivative when $v=v_j$, then we have with the same degree of accuracy

$$r-r_0=\frac{\gamma+1}{2}(v-v_J)(t-t_0)\qquad\left(r_0=\Phi(v_J),\quad t_0=-\frac{2}{\gamma+1}\Phi'(v_J)\right) \tag{17}$$

i.e. the wave may be considered as being centralized. Taking the value of $v-v_J$ from (17) with $r=v_{10}\tau_0$, we write expression (16) in the following form:

$$\frac{\Delta^2}{2(\gamma+1)}\left[1-\left(\frac{t_0}{t-t_0}\right)^2\right] \tag{18}$$

Equation (14), after replacement $\Delta F_{*f,t}$ by expression (18), provides ready means for defining function F, and also of other looked for functions in the band comprised between the two fronts, step by step in each of the regions separated from each other by the characteristics of various sets. The magnitude of stream perturbations of the order Δ^2 downstream of the heat release front, superimposed on the approaching simple wave, may, if necessary, be determined from conditions (12).

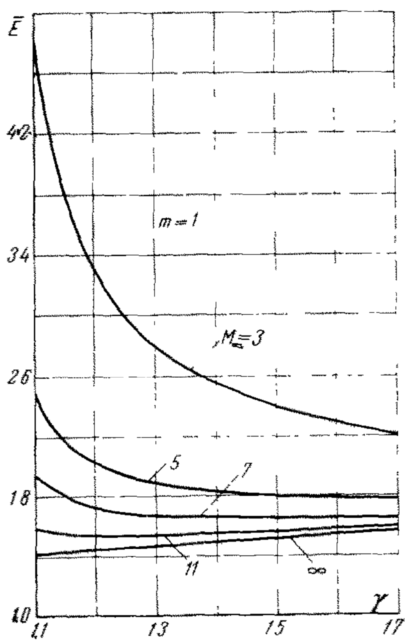

Fig. 4

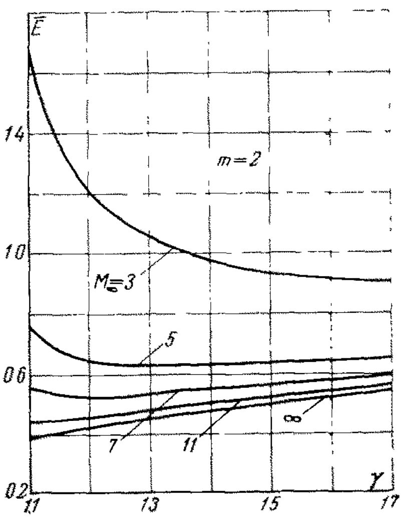

Fig. 5

We shall analyze the asymptotic behavior of the solution of Eq. (14) for $t\to\infty$, and

assume that
$$F_{f,\,t} = \Delta^2 (c + c_1 e^{zt/\tau_0}) \quad (z = \beta + i\Omega)$$

Here β and Ω are real numbers. Substituting this expression into Eq. (14), we obtain
$$c = \frac{k}{2(\gamma + 1)(1 - \lambda' - \lambda'')}$$

After certain transformations we obtain from this
$$\frac{(c_f)_\infty}{v_\infty} = \frac{(1 + \gamma M_\infty{}^2)^2}{2(\gamma + 1)^2 (M_\infty{}^2 + 1)(M_\infty{}^2 - 1)} \Delta^2 \tag{19}$$

It is easy to establish from the first of Eqs. (2) and the definition of Δ that the detonation wave velocity determined by this formula corresponds to the Chapman-Jouguet mode.

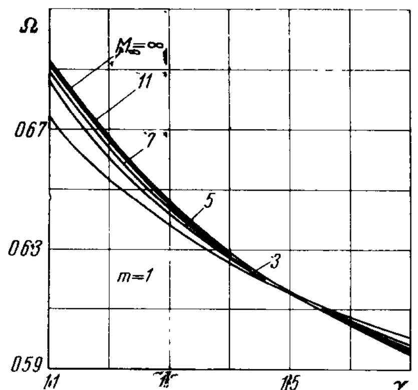

Fig. 6

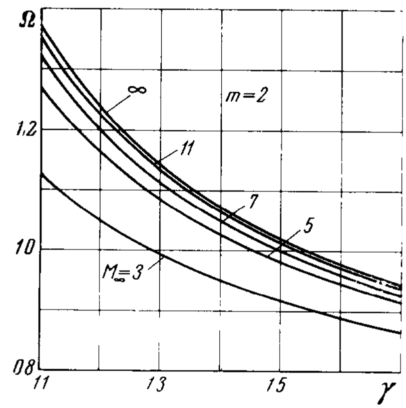

Fig. 7

The complex constant z appearing in the solution of the homogeneous equation of F must satisfy condition
$$\lambda' \exp \frac{-\delta_1 z}{\tau_0} + \left(\lambda'' + \frac{M_{10}}{1 - M_{10}} \lambda''' z\right) \exp \frac{-\delta_2 z}{\tau_0} - 1 = 0$$

For the solution to be stable it is necessary that all zeros appearing in the left side of this equality lie to the left of the imaginary axis of the plane of the complex variable z.

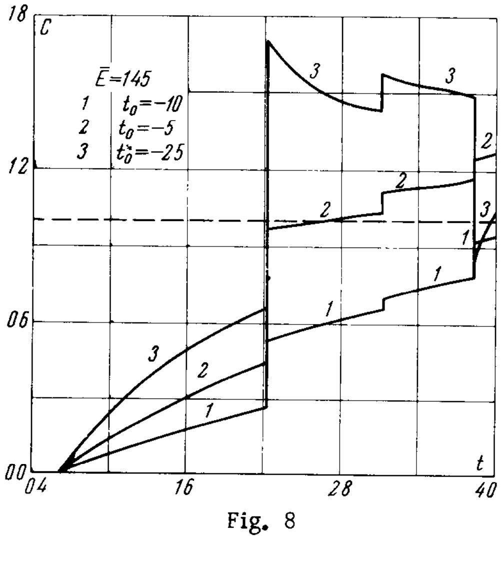

Fig. 8

The stability boundary is shown in Figs. 4 and 5 in terms of dependence of E/RT_{10} on γ and M_∞ along this boundary. Figs. 6 and 7 show the corresponding values of the neutral oscillation frequencies Ω. The stability boundary in the form of dependence of limit values of E/RT_∞ on M_∞ for $\gamma = 1.4$ is also shown on Fig. 3 by a dotted line. It follows from these data that the loss of stability at high M_∞ calculated on the basis of the linear mechanism, occurs at a lower activation energy than that obtaining with the nonlinear mechanism. In other words, if for certain values of the activation energy and heat release a detonation wave is stable, then in the course of its gradual weakening it will either remain stable, or begins to loose its stability in conformity with the linear mechanism.

We note, once again, that the problem of the two-front detonation wave weakened by a rarefaction wave approaching it from behind, as well as that of transition of detonation to the Chapman-Jouguet pattern, were previously considered in [3], where conditions (3)—(5) were presented in the approximate form

$$r_f = r_s + (v_s - c_s)\tau, \qquad \tau = \frac{k}{p_s^{m-1}} \exp\frac{E}{RT_s}$$

in which all magnitudes relate to one and the same instant of time t. Values of gas parameters downstream of the adiabatic jump and in front of the heat release front were assumed to be the same for a given t. With these assumptions, and in the presence of interaction with a sufficiently strong Riemann wave, the detonation wave assumes the Chapman-Jouguet pattern. However, this mode is stable with respect to small perturbations of the initial state (smooth exponential attenuation of perturbations) only (when $M_\infty = \infty$) for

$$\frac{E}{RT_{10}} < \frac{2\gamma}{3\gamma - 1} + m - 1 \qquad (20)$$

In the opposite case the perturbations increase exponentially with time.

We note that in the approximation considered in [3] the wave-like character of the perturbation buildup at loss of stability was not taken into consideration.

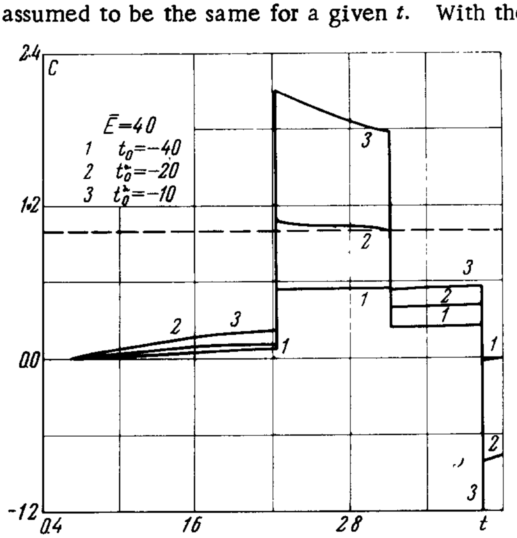

Fig. 9

We shall show in conclusion that according to the results of the present investigation the interaction of a two-front detonation wave with a rarefaction wave approaching it from behind may lead to oscillation onset also in the flow stability zone. To prove this we use Eq. (14) by substituting in it expression (18) for $\Delta F_{*f,t}$, and pass from function F to function c_s, selecting τ_0 as the unit of time and the Chapman-Jouguet wave velocity as the unit for c_s. As the result we obtain

$$c_t = \lambda' c_{t-\delta_1} + \lambda'' c_{t-\delta_2} + \lambda''' \frac{M_{10}}{1 - M_{10}} c'_{t-\delta_2} + (1 - \lambda' - \lambda'')\left[1 - \left(\frac{t_0}{t - \delta_0 - t_0}\right)^2\right]$$

$$\delta_0 = \frac{M_{10}}{1 - M_{10}} \tau_0 \quad (\delta_0 \text{ is a dimensional magnitude})$$

Some of the results of the adiabatic jump velocity calculations are shown on Figs. 8 and 9 for various values of parameter t_0 which characterizes the intensity of the rarefaction wave approaching the combustion front.

The conclusions reached in this work are in accord with the results of the numerical solution of the nonlinear problem presented in [6].

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STATIONARY CONVECTION IN A VERTICAL CHANNEL WITH PERMEABLE BOUNDARIES

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The problem of stationary heat convection in an infinitely long vertical flat channel with permeable boundaries is considered. The fluid is heated from below, so that in the channel there exists a constant temperature gradient. The fluid is blown into the channel through one of its vertical boundaries, and is sucked away through the other creating a transverse flow through the layer at a constant velocity. An exact solution of the problem of superposition of vertical convection on the homogeneous transverse flow is derived. Two kinds of motion are analyzed, viz. a plane, and a space motion which along the layer boundary depend periodically on the horizontal coordinate. It is shown that plane convection motions are only possible up to a certain limit of the fluid blowing-in rate.

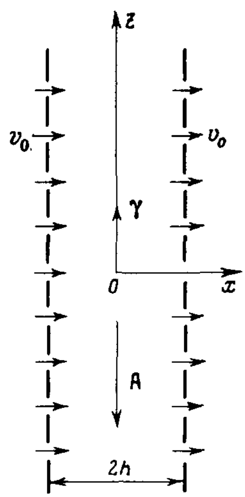

Fig. 1

1. A vertical plane layer of fluid is bounded by two parallel permeable planes $x = \pm h$ (Fig. 1). A fluid is uniformly blown into the channel through one of its boundaries at constant velocity v_0 and extracted through the other at the same uniform rate.

The heating from below generates in the fluid a vertical temperature gradient A directed downwards.

The equations of stationary convection are of the form [1]